Finding Quantities from Probabilities

Golden Retrievers have a baseline lifespan that follows a Normal model: $\text{Norm}(12, 1.5)$ in years. You are an actuary adjusting insurance policy thresholds. Your goal is to pinpoint the exact age cutoff x such that no more than 1% of these animals are expected to pass away before reaching it.

1. Sketch a bell curve representing this population distribution below. Mark the mean μ at the center line, and shade the region in the left tail representing 1% of the population.

2. **The Tool Trap:** To find this boundary on the horizontal axis, you reach for `pnorm()`. Explain why you hit a logical wall. What does `pnorm()` calculate, and what is missing in your attempt to solve for x?

Sally the Statypus says: `pnorm()` is an 'Area Machine.' It demands a boundary wall (x) to tell you how much area is shaded. If you don't have the wall, the machine cannot work backward! We need the inverse: a function that takes the area and finds the wall.

Bill the Statypus says: Look in Section 7.4 of your textbook to find a new function.

Coding Corner: Executing the Inverse Function

3. Now that you have reviewed the syntax rules in Section 7.4, write down and run the clean Base R command to calculate the exact age milestone for your insurance policy limits.

   R Code Blueprint: ____

   Calculated Age Cutoff: ______________________ years

Chapter 7

Prescriptive Engineering: Cost vs. Risk

You are an engineer for a company that produces titanium rods. Your customer demands that no more than 1-in-a-million (10^{-6}) rods can fall below a strength of 50 kN. You know that the strength of a rod is heavily correlated to the purity of the raw material used. However, you also know that purer raw materials can cost significantly more. This is going to be a very large and long running contract with an emerging aerospace company, so you purchase enough titanium at 3 different grades and find the sample mean ($\bar{x}$) and standard deviation (s). Luckily, since the samples are large, we can use $\mu \approx \bar{x}$ and $\sigma \approx s$.

Sally the Statypus says: Don't worry if the fancy word "correlate" isn't crystal clear! We will learn that term in the next chapter.

Bill the Statypus says: Yeah, for now it simply means that we know that purer materials make stronger rods.

Sally the Statypus says: Exactly! Ok, so below is the results of your 3 samples. We need to figure out what option we should start manufacturing.

Material Level	**Mean ($\bar{x}$)**	**Std Dev (s)**	**Cost / Rod**
Grade A	55.0 kN	1.30 kN	$12.00
Grade B	58.5 kN	1.45 kN	$17.50
Grade C	62.0 kN	1.80 kN	$32.00

1. **The Quantile Scan:** For each grade, use `qnorm()` to find the strength x achieved at the 1-in-a-million extreme low boundary ($p = 0.000001$).

   Grade A (x): ____________________ kN

   Grade B (x): ____________________ kN

   Grade C (x): ____________________ kN

2. **Managerial Decision:** Review the 50 kN safety requirement. Which grades of titanium are pure enough to meet the needs of the customer?

Seneca the Statypus: The Cost of Testing

While we quickly glossed over the concept of testing large samples of titanium rods on the previous page, it is important to remind ourselves how we test the strength of a manufactured part: We break it. We literally place increasing stress on the part until it breaks! This can be costly and why it is helpful to try and minimize the amount of costs are spent for just testing purposes.

Bill the Statypus says: Hey, Seneca, it has been a long time. You are of course right. This is often the cost of business and we would be expected to test, and thus destroy, a representative sample of the rods we do manufacture for the customer.

Sally the Statypus says: This is vital as we may have concerns that the strength may be affected by seasonal changes at our factory. Thus we really need to actually text each batch of rods that we make.

Bill the Statypus says: Yep, the good news is that the broken rods can be melted down and then used for products that don't require as much strength as the purity does go down.

Reflection: The Reality of Manufacturing

1. **The Testing Reality:** Your client wants 99.9999% of all rods to exceed 50 kN. To prove this, the manager you despise suggests you should test 1,000,000 rods to see how many break. Explain why this is not just impractical, but fundamentally irrational for a manufacturing business. (Don't forget **how** we test the rods.)

2. **Accounting for Uncertainty:** Your in-house statistician warns you: "Because we are using s (sample standard deviation) to estimate σ (population standard deviation), our estimate has a small amount of error. If we are aiming for such high reliability, we shouldn't just use the raw threshold we calculated. We should build in a 'safety margin' of error." Explain why adding this margin makes the company safer, even if it might mean rejecting a grade of titanium that seemed effective by our base calculation.

3. **The Final Recommendation:** Review your quantile results from Page 2 and consider the cost of each grade. Write a brief recommendation to the client:
   - Which material grade do you recommend?
   - Why is this choice the "smartest" bet for the company? (Consider the cost of the material vs. the risk of failing the safety contract).

   Write your responses on the next page.

Write your responses to the reflection questions here.

Chapter 7

Sally the Statypus says: Wow, this problem seemed so straightforward! However, now, I am not even sure which grade of titanium I would choose.

Bill the Statypus says: Sally, most textbooks ask you questions where there is a definite answer. In the real world, there is rarely such a clear result. Reality asks us to balance cost, safety, and many other factors at all time. Statistics doesn't always give you "the answer," but instead it offers you tools to understand the situation and pick the best option you can.

Sally the Statypus says: Reality is messy! Maybe the classroom isn't such a bad place after all.