Business Under Random Pressure

An exclusive dinner theater has exactly 50 physical seats available. To maximize their revenue on a Friday night performance, the management accepts 55 total seat reservations. From comprehensive archival data, the house manager knows that any single reservation has a constant, independent probability of $p = 0.85$ of actually showing up.

Sally the Statypus says: This is a high-stakes balancing act. If fewer than 50 parties show up, the venue loses out on potential drink and dinner sales. But if more than 50 show up, they may have an absolute customer service nightmare on their hands.

Bill the Statypus says: Before we throw this at our R engine, let's look at the information. Our random variable X is the total count of parties that show up for the performance. This seems to fit our Binomial criteria perfectly: a fixed maximum, independent parties, and a binary outcome.

Reflection: Task 1: Setting up the Constraints

1. Identify the arguments required to parameterize our distribution model, and translate the text logic into exact mathematical shorthand.

   a. Define the parameter values for this model: $n =$ __________ $p =$ __________

   b. We want to find the probability that a customer service disaster occurs (the venue runs out of physical seats). Write this statement out in raw mathematical probability notation:

   c. We won't discount the model at this time, but describe some reasons why the binomial model may not be ideal in this scenario

2. What is the probability that the restaurant will have more than 50 people show up on Friday night if they accept 55 seat reservations? Give both the code used and the numerical probability found from R.

Statypus Insight: The Business Reality

Overbooking a dining room can have a negative impact on reviews and public opinion. However, empty seats lower our profits. Restaurants may choose to accept some risk to increase profits.

Let's downscale our business logic to where we can see the under-the-hood arithmetic of long-term patterns. A small bistro patio has exactly $n = 5$ outdoor tables. The owner tracks how many tables are occupied during the lunch rush. Let the random variable X represent the total count of occupied tables.

Bill the Statypus says: Do not let the greek letters and math shorthand scare you! The expected value of a discrete random variable is simply the long-run average value we would see if we repeated the afternoon shift thousands of times. The formula looks like this:

$$\mu_X = E[X] = \sum x \cdot P(X = x)$$

This means that you take each possible outcome, multiply it by its probability, and add them up!

3. Complete the table below to find the long-term expected number of occupied tables on the patio. For the values of $P(X = x)$), assume the number of tables can be modeled with $\texttt{binom}(5, 0.75)$.

Tables Occupied (x)	**Probability** ($P(X = x)$)	**Product** ($x \cdot P(X = x)$)
0		
1		
2		
3		
4		
5		
Total Sum ($\sum$)	1.00	**Expected Value:**

4. If the restaurant takes in an average of \$100 per used table, what is the expected revenue brought in during an afternoon shift.

Sally the Statypus says: Notice that a full restaurant would bring in an average of \$500 for an afternoon shift. This shows the mentality of why places may consider overbooking.

Chapter 8