The Discrete Disconnect

Statypus Insight: Counting versus Measuring

The English language can sometimes be your biggest mathematical hurdle. Phrases like "at least," "fewer than," and "no more than" must be translated into strict ($<, >$) or non-strict ($\leq, \geq$) inequalities with absolute precision before you write a single line of R code.

Read Section 8.1 in your textbook before moving forward. The video shown there is a masterclass in how the Binomial distribution can be used to help us make a decision we likely face quite often.

Sally the Statypus says: I remember this exact transition tripping me up when I first learned it. In Chapter 7, we were told that $P(X \geq 10)$ and $P(X > 10)$ were practically the exact same thing. Then, suddenly, code will give you the wrong answer if you mix them up!

Bill the Statypus says: It is a hard pivot! In Chapter 7, we were *measuring continuous* things, like the mass of a platypus. The probability of an animal being exactly 2.000... kg was zero. But here in Chapter 8, we are *counting discrete* things, like people or correct answers. The probability of exactly 10 has a real weight! So, "more than 10" means we have to start counting at 11.

Sally the Statypus says: Head to your textbook and look up the syntax of the functions `dbinom()` and `pbinom()`. This will help you fill out the table below.

1. For each English phrase below, write the raw Probability Notation. Then, rewrite it to match one of the two formats `pbinom` accepts ($X \leq q$ or $X > q$). Finally, identify the `q` value and the `lower.tail` setting you would use in *R*.

English Phrase	**Raw Notation**	`pbinom` **Format** ($X \leq q$ or $X > q$)	`q` **Value**	`lower.tail?`
"At least 7"	$P(X \geq 7)$	$P(X > 6)$	6	`FALSE`
"No more than 12"				
"Fewer than 5"				
"More than 9"				
"Exactly 8"		*(Hint: You do not use* `pbinom` *for this. What do you use?)*		

Statypus Insight: The Code Connect

Note that in Chapter 6 we had the functions `pdist` and `qdist` which then became `pnorm` and `qnorm` when we moved to the Normal distribution. We used `dist` for a generic **dist**ribution and then moved to `norm` for the **Norm**al distribution. We now use `binom` for the **binom**ial distribution.

2. If a student guesses randomly on a True/False quiz (so $p = 0.5$) which has 10 questions (so $n = 10$), find the probability that the student will get at least 7 correct in order to pass. Give both the code used and the numerical probability found from R.

3. If instead there were 20 questions on the quiz (so $n = 20$) and they still have a 50/50 chance at each answer, find the probability that the student will get at least 12 but no more than 17 correct answers. Give both the code used and the numerical probability found from R.

Sally the Statypus says: This one can be tricky! I would look at Example 8.4 in your book.

Bill the Statypus says: Just like in Chapter 7, we can also find a quantile (or quantity) given a set probability (or proportion). We just need to use the `q` function, which is now `qbinom`.

4. If a student does indeed guess randomly on the 20 question True/False quiz, what is the minimum score they can expect to get at least 75% of the time.

Sally the Statypus says: This one is also tricky! We are given that $p = 0.75$, but we must put that in the upper tail, I mean set `lower.tail` to be `FALSE`. Give both the code used and the numerical probability found from R.

Seneca the Statypus: A full bag of p's

It can be tricky dealing with 2 different uses of the letter `p`. In the textbook, we use p for the theoretical probability of each Bernoulli trial but record it as `prob` within the R functions. The value of `p` in `qnorm` is the probability (or proportion) that we want in the tails of the distribution.

8.3: Binomial or Bust?

To use the Binomial Distribution, we need a setup with n independent trials, each with two possible outcomes ("Success" or "Failure") and a constant probability of success (p). If any of these conditions break, the model breaks.

Example 1: The Binomial Case *You drive to work every day and encounter a specific traffic light. Let's assume the probability of getting caught at this light is 30%. We track this for 20 work days.*

Bill the Statypus says: This is a great Binomial scenario! We have a fixed $n = 20$, the outcome is binary (caught or not caught), and it is reasonable to assume that getting caught at the light on Tuesday is independent of getting caught on Wednesday. The probability $p = 0.30$ remains constant for each day.

Example 2: The Non-Binomial Case *A researcher studies the rate of a highly contagious respiratory disease in a communal group of animals living in a single, confined pen.*

Sally the Statypus says: This one fails the "independence" check. Because the animals share a confined space, if one gets sick, the probability of the others getting sick skyrockets. The trials are dependent, so the Binomial model would not accurately model the spread of the disease.

Bill the Statypus says: This doesn't mean there isn't a model for this scenario, it only means that the binomial model is not well suited.

Chapter 8

Reflection: Diagnose the Scenario

For each scenario below, determine if it can be modeled using a Binomial Distribution. Explain why or why not, focusing on which (if any) condition is violated.

1. **The Commute Count:** You track the total number of traffic lights you get caught at during your daily commute across 20 days. (Note: You drive through 5 different lights each day).
2. **The Game of Trouble:** In the game of "Trouble," you repeatedly press the Pop-O-Matic bubble to try and get a 6. You track how times you get a 6 over a long game.

Statypus Insight: The Independence Trap

Remember: The Binomial model is an approximation. In the real world, absolute independence is rare (e.g., family members at a concert likely decide together). We use the Binomial model because it gives us a powerful estimate, not because it is a perfect mirror of reality.

Record your answers to the reflection questions on the next page.

Record your answers to the reflection questions here.

Bill the Statypus says: Well, I sure had fun with this one. You can *count* on that!

Sally the Statypus says: Oh, Bill...

Bill the Statypus says: (Chuckling) What?!