Continuous Distributions: The Density Perspective

Bill the Statypus says: If you try to count every grain of sand on a beach, you will be here forever. Some things in nature are better measured than counted. When we move to continuous variables, the probability of any *exact* value is effectively zero, so we shift our focus from discrete counts to **density** and **area**.

Seneca the Statypus: The Continuous Variable Axiom

For any continuous random variable X, the probability of X taking on any **exact** value is zero:

$$P(X = x) = 0$$

Instead, we calculate probabilities over an **interval**, defined as the area under the density curve between two points:

$$P(a < X < b) = \text{Area under the curve between } a \text{ and } b$$

Sally the Statypus says: An important implication of the Continuous Variable Axiom is that we can replace $<$ with $\leq$ in one or both of the places it appears in $P(a < X < b)$ without changing its value! This is **NOT** true for a discrete random variable!

1. The Four R Functions

Statypus Insight: Function Architecture

For any continuous distribution in R, we use four fundamental algorithmic functions. Memorizing these is the key to unlocking simulation:

ddist: Density (Height)	**p**dist: Probability (Area to left)
qdist: Quantile (Value)	**r**dist: Random (Generator)

Bill the Statypus says: The concept of a **Density Function** can be confusing for most new statyp... I mean human students. Please review section 6.5.1 in r.statypus.org to gain an understanding of what a density function means.

Sally the Statypus says: Bill, we love all of our students, whether they have webbed feet or not. One important thing to remember is that individual values of a density function are not probabilities. We can only get probabilities by looking at the areas under the curve.

Bill the Statypus says: Correct! The probability of a continuous random variable is defined to be the definite int...

Sally the Statypus says: Bill! We don't use the C word in this class! Don't mind him... let's move on!

Statypus Insight: Function Architecture

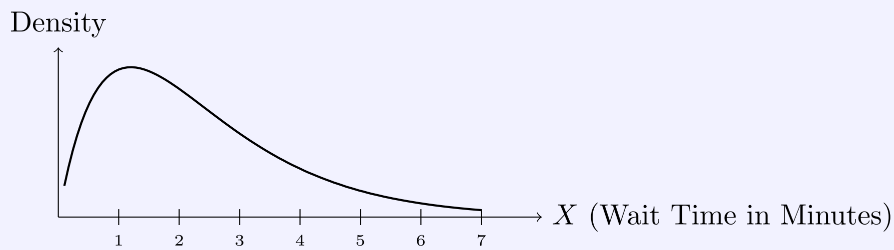
For any continuous distribution in R, we use three fundamental keys. Look these up in the **New R Functions Used** section of your textbook:

pdist: Probability (Area to left) **q**dist: Quantile (Value)

rdist: Random (Generator)

Reflection: Friction Block: Sketching Density

Below is a smooth, right-skewed density curve representing the wait time (in **minutes**) for a fresh scone at the Statypus Cafe.



1. **p-dist:** Shade the area under the curve to the left of $x = 2$. Label this area as the output of the `pdist()` function. What does this value represent in terms of the probability of the wait time?

2. **q-dist:** If you want to find the wait time that defines the 90th percentile, you are looking for a coordinate on the X axis. Explain why the `qdist()` function is the mathematical inverse of the `pdist()` function.