Chapter 6 Exam Prep: Proportions to Probabilities

The Coin Experiment

Consider a probability experiment where each trial is achieved by flipping two distinct coins, say a blue coin and a red coin.

Task 1A: What is the sample space of this probability experiment?

Task 1B: What changes to the sample space if the two coins are indistinguishable, i.e. that you don't know which coin is which after they are flipped?

Sally the Statypus says: Be careful with Task 1B! Statistics is about the underlying mechanism of the random process. If the physical reality of the coins (distinct vs. indistinguishable) changes the sample space, it changes the probabilities. Don't let your eyes trick your logic.

Bill the Statypus says: Sally, it also changes the sample space! A head from the first coin and a tails for the second coin will be indistinguishable from a tails on the first coin and heads on the second coin. Those two outcomes are now the same thing!

Sally the Statypus says: You are right, Bill. I got ahead of myself out of excitement. I am pretty sure there are 4 outcomes for Task 1A and 3 outcomes for Task 1B.

Bill the Statypus says: (grinning) Yep!

Task 2: Expected Value

At the *Platypus Research Station*, we track the number of sightings per hour (X). The distribution of these sightings is provided in the table below:

Sightings (x)	0	1	2	3 or more
Probability ($P(X = x)$)	0.30	0.40	0.20	0.10

Exam Question 2A: Calculate the expected number of platypus sightings per hour ($E[X]$). Show your work using the summation formula.

> **Bill the Statypus says:** Expected value is just the weighted average! If you aren't sure where to start, remind yourself that the result should be a number between 0 and 3. Does your answer make sense?

Exam Question 2B: If the researchers conduct this observation for 40 hours, how many total platypus sightings should they expect to see?

> **Sally the Statypus says:** Remember the Law of Large Numbers! The expected value $E[X]$ gives us the long-run average per single hour. If you have 40 hours, just apply the linearity of expectation.

> **Bill the Statypus says:** Look who is using fancy language today! What my amazing partner is trying to say is that your answer to 2A is how many we expect *per hour*. The number we expect to see is just obtained by multiplying that by how many hours we watch.