Chapter 10 Exam Prep: One-Sample Inference

Statypus Insight: The Precision Upgrade

Because we rarely know the true population standard deviation (σ), we must rely on our sample's standard deviation (s) to calculate the Standard Error. This extra level of uncertainty means we can no longer use the standard Normal curve. Instead, we upgrade to the **t-distribution**, which has heavier tails to account for the extra uncertainty!

Raw Exam Question & Output:
Historical records suggest that the mean depth of platypus nesting burrows in the Eucumbene River is 2.5 meters. Researchers suspect that recent changes in water levels have caused the platypuses to dig deeper or shallower burrows. They collect a random sample of $n = 35$ active burrows to test if the mean depth is now significantly different from 2.5 meters.

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> t.test(BurrowDepth, mu = 2.5, alternative = "two.sided")

One Sample t-test

data:  BurrowDepth
t = 2.218, df = 34, p-value = 0.0333
alternative hypothesis: true mean is not equal to 2.5
95 percent confidence interval:
 2.5252  3.0748
sample estimates:
mean of x
      2.8
```

Exam Tasks (Part 1):

A. State the null and alternative hypotheses (H_0 and H_a) using the correct notation.

B. Based on the R output, state your formal conclusion at the $\alpha = 0.05$ significance level.

Sally the Statypus says: Notation Trap! We are testing an average, not a percentage. Do not use p or $\hat{p}$ here. Also, remember that hypotheses are ALWAYS written about the true population parameter (μ), never the sample statistic ($\bar{x}$).

Bill the Statypus says: Parsing friction! The computer did the heavy lifting, but you have to find the signal. Locate the p-value in the text wall. Is it less than 0.05? If so, you have enough evidence to reject the historical average!

Your Turn (Record your answer on the blank portion on the top of the next page.)

Your Turn (Answers for Part 1):

Exam Tasks (Part 2):

C. Locate the 95% Confidence Interval in the R output on the previous page. Write a formal sentence interpreting this interval in the context of the burrow depths.

Sally the Statypus says: Fill in the blanks! The skeleton structure for means is the exact same as it was for proportions: "We are [XX]% confident that the true [parameter] of [population context] is between [lower bound] and [upper bound] [units]." Do not forget your units (meters)!

Seneca the Statypus: The Consistency Check

Even though Hypothesis Tests and Confidence Intervals are tested separately, they are two sides of the same logical coin.

Look at your decision from Part B: Because the p-value was 0.0333, you rejected the idea that the true mean was 2.5 meters. Now look at your interval from Part C: $[2.5252, 3.0748]$.

Notice that 2.5 is **not** inside that interval! The math and the logic perfectly support each other. If you claim the mean is not 2.5, your interval shouldn't contain 2.5 either.